
AP[®] Calculus AB

Sample Student Responses and Scoring Commentary

Inside:

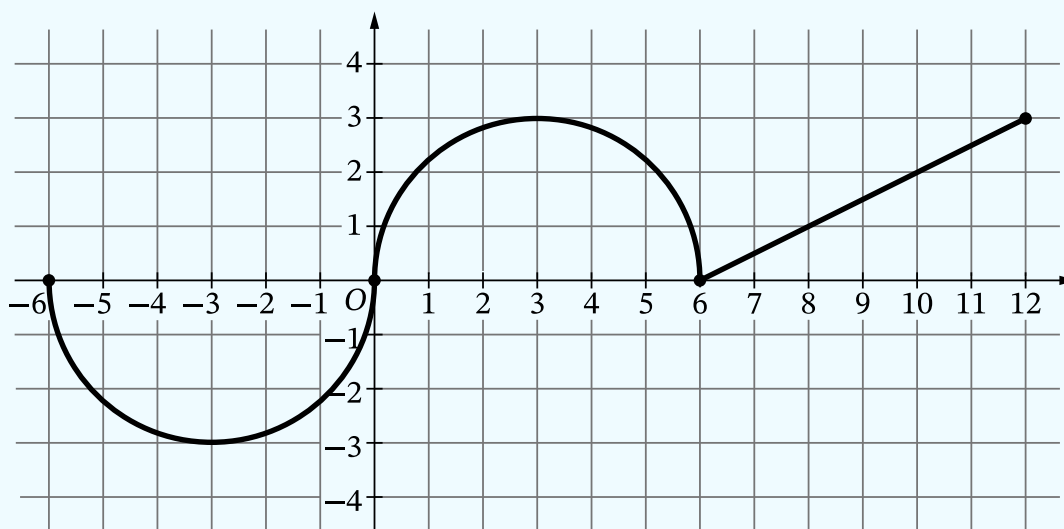
Free-Response Question 4

- ☒ **Scoring Guidelines**
- ☒ **Student Samples**
- ☒ **Scoring Commentary**

Part A (AB or BC): Graphing calculator not allowed**Question 4****9 points****General Scoring Notes**

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

The continuous function f is defined on the closed interval $-6 \leq x \leq 12$. The graph of f , consisting of two semicircles and one line segment, is shown in the figure.

Graph of f

Let g be the function defined by $g(x) = \int_6^x f(t) \, dt$.

Model Solution		Scoring
A	Find $g'(8)$. Give a reason for your answer.	
	$g'(x) = f(x)$	Considers $g'(x) = f(x)$ Point 1 (P1)
	$g'(8) = f(8) = 1$	Answer Point 2 (P2)
Scoring Notes for Part A		
P1 is earned for $g' = f$, $g'(x) = f(x)$, or $g'(8) = f(8)$ in part A. - A response of $g'(8) = f(8) = 1$ earns both P1 and P2. - A response that does not earn P1 can earn P2 with an implied application of the Fundamental Theorem of Calculus (e.g., $g'(8) = 1$ or $f(8) = 1$). - A response of $g'(8) = f(8) - f(6) = 1$ earns P2 but not P1.		

- B** Find all values of x in the open interval $-6 < x < 12$ at which the graph of g has a point of inflection. Give a reason for your answer.

The graph of g has a point of inflection where $g'' = f'$ changes sign, which is where $g' = f$ changes from decreasing to increasing or vice versa.

The graph of g has points of inflection at $x = -3$ and $x = 6$ because f changes from decreasing to increasing there.

The graph of g also has a point of inflection at $x = 3$ because f changes from increasing to decreasing there.

Answer **Point 3 (P3)**

Reason **Point 4 (P4)**

Scoring Notes for Part B

- **P3** is earned only for an answer of $x = -3$, $x = 3$, and $x = 6$. If any other/additional values of x in $-6 < x < 12$ are declared to be points of inflection, the response does not earn either **P3** or **P4**. Consideration of $x = -6$ or of $x = 12$ does not impact scoring.
- To earn **P4**, a response must tie the reason to the given graph of f .
 - A response of “ g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because f changes from increasing to decreasing or decreasing to increasing there” earns both **P3** and **P4**.
 - A response of “ g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because the slope of f changes sign there” earns both **P3** and **P4**.
 - A response of “ g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because f attains relative extrema there” earns both **P3** and **P4**.
 - A response of “ g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because g changes concavity there” earns **P3** but not **P4**.
 - A response of “ g has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because $g'' = f'$ changes sign there” earns **P3** but not **P4**.
 - A response that relies upon an ambiguous term such as “the function” or “the graph” does not earn **P4**.
- **Special case:** A response with two of the three correct x -values with correct reasoning and no other/additional values of x declared to be points of inflection earns **P4** but not **P3**.

C Find $g(12)$ and $g(0)$. Label your answers.

$$g(12) = \int_6^{12} f(t) \, dt = \frac{1}{2} \cdot 6 \cdot 3 = 9$$

 $g(12)$ **Point 5 (P5)**

$$g(0) = \int_6^0 f(x) \, dx = -\int_0^6 f(x) \, dx = -\frac{\pi}{2} 3^2 = -\frac{9\pi}{2}$$

 $g(0)$ **Point 6 (P6)****Scoring Notes for Part C**

- Unlabeled values do not earn either **P5** or **P6**.
- **P5** is earned for a response of $g(12) = 9$, with or without supporting work.
- **P6** is earned for a response of $g(0) = -\frac{9\pi}{2}$, with or without supporting work.

Note: Incorrect communication between the label “ $g(0)$ ” and the answer will be treated as scratch work and will not impact scoring. For example, $g(0) = \int_0^6 f(x) \, dx = -\frac{9\pi}{2}$ earns **P6**.

- D** Find the value of x at which g attains an absolute minimum on the closed interval $-6 \leq x \leq 12$. Justify your answer.

For $-6 \leq x \leq 12$, g attains a minimum either when $g'(x) = f(x) = 0$ or at an endpoint.

$$g'(x) = f(x) = 0$$

$$\Rightarrow x = 0, x = 6$$

x	$g(x)$
-6	0
0	$-\frac{9\pi}{2}$
6	0
12	9

Therefore, on the closed interval $-6 \leq x \leq 12$, g attains an absolute minimum value at $x = 0$.

Considers $g'(x) = 0$ **Point 7 (P7)**

Justification **Point 8 (P8)**

Answer **Point 9 (P9)**

Scoring Notes for Part D

- P7** is earned for considering $g'(x) = 0$ or $f(x) = 0$. **P7** is not earned by just presenting $x = 0$ and $x = 6$.
A response that discusses the sign of $g'(x)$ or $f(x)$ changing OR uses the phrase “critical points of g ” also earns **P7**.
- To earn **P8** using a candidates test, a response must make a global argument by providing evaluations or reasoning for each of $g(-6)$, $g(0)$, $g(6)$, and $g(12)$ (and no other x -values).
- Alternate justification and answer:
Because $g'(x) \leq 0$ (or $f(x) \leq 0$) for $-6 \leq x < 0$ and $g'(x) \geq 0$ (or $f(x) \geq 0$) for $0 < x \leq 12$, the absolute minimum of g occurs at $x = 0$.
- A response that presents a local argument (such as a First Derivative Test or a Second Derivative Test) or an incorrect global argument does not earn **P8** but is eligible for **P9** with the correct answer of $x = 0$.
- For **P8**, values of $g(0)$ and $g(12)$ can be imported from part C. A response can earn **P9** with an answer that is consistent with the imported values.

Q4

NO CALCULATOR ALLOWED

Q4

Answer QUESTION 4 PARTS A and B on this page.

PART A

$$\frac{d}{dx}g(x) = \frac{d}{dx} \int_0^x f(t) dt$$

$$g'(x) = f(x)$$

$$g'(8) = f(8) = \boxed{1}$$

fundamental theorem of calculus
states that $\frac{d}{dx} \left[\int f(x) dx \right] = f(x)$

PART B

g has a pt of inflection at $x = -3$ because f (or g') changes from decreasing to increasing

g has a pt of inflection at $x = 3$ because f (or g') changes from increasing to decreasing

g has a pt of inflection $x = 6$ because f (or g') changes from decreasing to increasing

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Q4



NO CALCULATOR ALLOWED

Q4

Answer QUESTION 4 PARTS C and D on this page.

PART C

$$g(12) = \int_6^{12} f(t) dt$$

$$g(12) = \frac{(6)(3)}{2}$$

$$g(12) = 9$$

$$g(0) = \int_6^0 f(t) dt$$

$$g(0) = - \int_0^6 f(t) dt$$

$$g(0) = - \left(\frac{1}{2} \pi (3)^2 \right)$$

$$g(0) = - \left(\frac{9}{2} \pi \right)$$

$$g(0) = - \frac{9}{2} \pi$$

PART D

g is continuous on $[-6, 12]$

$\therefore$ EVT applies

$$g'(x) = f(x)$$

$$0 = f(x)$$

$$x = 0 \quad x = 6$$

x	$g(x)$
-6	0
0	$-\frac{9}{2}\pi$
6	0
12	9

$$g(0) < g(-6) = g(6) < g(12)$$

$\therefore g$ attains an absolute minimum value at $x = 0$

Q4



NO CALCULATOR ALLOWED

Q4

Answer QUESTION 4 PARTS A and B on this page.

PART A

$$g'(x) = f(x)$$

$$g'(8) = f(8)$$

$$g'(8) = 1$$

$f(8)$ appears to
be 1 when
looking at
the graph

PART B

when $x=0$ there appears
to be a point of inflection
as f goes from concave up to
concave down.

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Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

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Q4

NO CALCULATOR ALLOWED

Q4

Answer QUESTION 4 PARTS C and D on this page.

PART C

$$g(12) = 9$$

$$g(0) = -\frac{9}{2}\pi$$

PART D

$$g'(x) = f(x) = 0$$

$$x = 0, 6$$

minimum when $x=0$
 knowing that $g'(x)=f(x)$
 using the FTC, 0 is
 a minimum as $f(0)=0$
 meaning the slope of g
 0 and f goes from negative
 to positive at that point making
 it a minimum.

Q4

NO CALCULATOR ALLOWED

Q4

Answer QUESTION 4 PARTS A and B on this page.

PART A

$$g(x) = \int_0^x f(t) dt$$

$$g'(x) = f(x)$$

$$g'(8) = f(8)$$

$$g'(8) = 1$$

$g'(8) = 1$ because $g'(x) = f(x)$, when $x = 8$, $f(8) = 1$ and because the derivative of $g(x)$ equals $f(x)$, $g'(8)$ will also equal 1.

PART B

$$g''(x) = f'(x)$$

$$(P \& I: x = 0, 6)$$

Because $g''(x) = f'(x)$, when the slope of $f(x)$ changes signs, $g(x)$ will have a point of inflection.

Q4

NO CALCULATOR ALLOWED

Q4

Answer QUESTION 4 PARTS C and D on this page.

PART C

$$g(x) = \int_6^x f(t) dt$$

$$g(12) = \int_6^{12} f(t) dt = 18$$

$$g(12) = 18$$

$$g(0) = \int_6^0 f(t) dt = -\int_0^6 f(t) dt$$

$$g(0) = -\frac{1}{2} \pi (3)^2$$

$$g(0) = -\frac{9\pi}{2}$$

PART D

$$g'(x) = f(x)$$

$$g(6) = 0$$

$$g(-6) = 0$$

$$g(12) = 18$$

g attains an absolute maximum

at $x = 12$ because $g(x) = \int_6^x f(t) dt$,
and at $x = 6$, the integral equals
12, so $g(6) = 12$.

Question 4

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

NEW for 2025: The question overviews can be found in the *Chief Reader Report on Student Responses on AP Central*.

Sample: 4A

Score: 9 (1-1-1-1-1-1-1-1-1)

The response earned 9 points: 2 points in part A, 2 points in part B, 2 points in part C, and 3 points in part D.

In part A the response earned **P1** with the equation $g'(x) = f(x)$ in line 2. The response earned **P2** with the statement $g'(8) = f(8) = 1$ in line 3. This statement alone would have earned both **P1** and **P2**. Note: The comment presented on the right regarding the Fundamental Theorem of Calculus was not considered in scoring **P1** or **P2**.

In part B the response earned **P3** with the presentation of the correct answer $x = -3$, $x = 3$, and $x = 6$, with no additional values presented. The response earned **P4** with the correct reasoning that “ f (or g') changes from decreasing to increasing” in the first and third sentences and “ f (or g') changes from increasing to decreasing” in the second sentence.

In part C the response earned **P5** with the boxed statement $g(12) = 9$ in line 3 on the left. The numerical expression $g(12) = \frac{(6)(3)}{2}$ in line 2 on the left would have earned the point with no simplification. The response earned **P6** with the boxed statement $g(0) = -\frac{9}{2}\pi$ in line 5 on the right. The numerical expression $g(0) = -\left(\frac{1}{2}\pi(3)^2\right)$ in line 3 on the right would have earned the point with no simplification.

In part D the response earned **P7** with the statement $0 = f(x)$ in line 4 on the left. The response earned **P8** with the presentation of the correct table on the right. The response earned **P9** for the boxed statement that “ g attains an absolute minimum value at $x = 0$ ” on the right.

Sample: 4B

Score: 6 (1-1-0-0-1-1-1-0-1)

The response earned 6 points: 2 points in part A, 0 points in part B, 2 points in part C, and 2 points in part D.

In part A the response earned **P1** with the equation $g'(x) = f(x)$ in line 1. The response also would have earned this point with the equation $g'(8) = f(8)$ in line 2. The response earned **P2** with the equation $g'(8) = 1$ in line 3.

In part B the response did not earn **P3** because the answer $x = 0$ in line 1 is incorrect. The response did not earn **P4** because the response does not have at least two of the three correct x -values and presents incorrect reasoning.

In part C the response earned **P5** with the correct labeled value of 9 for $g(12)$ in line 1. The response earned **P6** with the correct labeled value of $-\frac{9}{2}\pi$ for $g(0)$ in line 2.

Question 4 (continued)

In part D the response earned **P7** with the equation $g'(x) = f(x) = 0$ in line 1. The response did not earn **P8** because a correct justification of an absolute minimum is not presented. The justification presented is not sufficient to establish that $x = 0$ is an absolute minimum on the given interval. The response earned **P9** for presenting the correct answer of $x = 0$ and a local argument that this is the location of a minimum.

Sample: 4C

Score: 3 (1-1-0-0-0-1-0-0-0)

The response earned 3 points: 2 points in part A, 0 points in part B, 1 point in part C, and 0 points in part D.

In part A the response earned **P1** with the equation $g'(x) = f(x)$ in line 2. The equation $g'(8) = f(8)$ in line 3 would also have earned the point. The response earned **P2** with the boxed statement $g'(8) = 1$ in line 4.

In part B the response did not earn **P3** because $x = 0$ is presented as an answer in line 2. The response did not earn **P4**. An incorrect value of $x = 0$ is presented, making the response ineligible for **P3** or **P4**.

In part C the response did not earn **P5**. An incorrect value for $g(12)$ is presented. The response earned **P6** with the correct labeled value of $-\frac{9\pi}{2}$ for $g(0)$ in the boxed response given in the last line. The numeric expression $g(0) = -\frac{1}{2}\pi(3)^2$ would have earned the point in line 5 with no further simplification.

In part D the response did not earn **P7**. No evidence is presented that shows consideration of $g'(x) = 0$ or $f(x) = 0$. The response did not earn **P8**. A complete candidates test is not presented, as $g(0)$ is not considered. The response is eligible for **P9**. The response did not earn **P9** because the correct location of the minimum is not presented.